Chapter 8 problems

Problem 8.1
A new type of nano-wire laser is to be designed with optical emission at $\lambda = 1550\ \text{nm}$. The laser consists of an InGaAs semiconductor active region with refractive index $n_{\text{InGaAs}} = 4$ inside a Fabry-Perot cavity of length L_C. The InGaAs is of width 40 nm and of thickness 40 nm. The sides of the wire are embedded in InP which has a refractive index of $n_{\text{InP}} = 3.22$. One mirror end of the wire is faced with gold of reflectivity 0.99 and the other end is bonded to sapphire (Al_2O_3) which has refractive index $n_{Al_2O_3} = 1.78$.

(a) Find the reflectivity of the mirror formed at the semiconductor sapphire interface.

(b) Estimate the optical confinement factor of the structure.

(c) Assume a spontaneous emission coefficient $\beta = 10^{-4}$, optical confinement factor $\Gamma = 0.1$, non-radiative recombination rate $A_{\text{nr}} = 10^8\ \text{s}^{-1}$, radiative recombination coefficient $B = 10^{-10}\ \text{cm}^3\ \text{s}^{-1}$, nonlinear recombination coefficient $C = 5 \times 10^{-29}\ \text{cm}^6\ \text{s}^{-1}$, a bulk gain model with optical transparency at carrier density $n_{\text{ot}} = 10^{18}\ \text{cm}^{-3}$, optical gain slope coefficient $G_{\text{slope}} = 2 \times 10^{-16}\ \text{cm}^2\ \text{s}^{-1}$, optical gain saturation coefficient $\varepsilon_{\text{bulk}} = 5 \times 10^{-18}\ \text{cm}^3$, and internal optical loss $\alpha_{\text{int}} = 10\ \text{cm}^{-1}$. Calculate the $L_{\text{out}} - I_{\text{inj}}$ characteristics and threshold current for a wire of length $L_C = 10\ \mu\text{m}$, $L_C = 30\ \mu\text{m}$, and $L_C = 100\ \mu\text{m}$.

(d) Discuss how to decrease laser threshold current.

Problem 8.2
Driving a laser with a short electrical pulse can generate an optical pulse.

(a) Using the laser diode described in Exercise 8.4, plot the full-width at half-maximum (FWHM) of laser optical pulse output as a function of electrical pulse width. The electrical pulse has a rectangular shape with maximum value $I_{\text{max}} = 50$ mA and minimum value $I_{\text{min}} = 0$ mA and width τ.

(b) What happens to the minimum optical pulse width if I_{min} is allowed to increase such that $I_{\text{max}} > I_{\text{min}} > 0$ mA?

(c) Find values of I_{min}, I_{max}, and τ that on average result in emission of one photon per pulse.

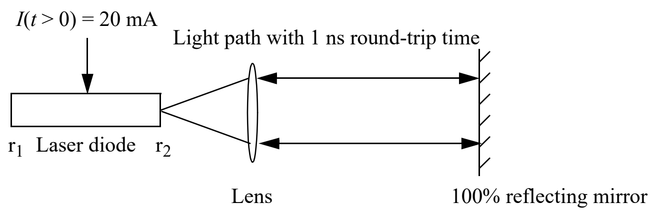
Problem 8.3
(a) Plot the noise-free transient and steady-state behavior for the laser diode of Exercise 8.4 but with each mirror having reflectivity 0.999, active region volume $2 \times 1 \times 0.2\ \mu\text{m}^3$ with cavity length $2\ \mu\text{m}$, and a step current of $100\ \mu\text{A}$.

(b) Repeat (a) only now in the presence of uncorrelated Gaussian photon and carrier number noise, i.e. use Eq. (8.82) and Eq. (8.83), but ignore Eq. (8.84).

(c) Use (b) to numerically find the mean and standard deviation of photon number for times long after the transient associated with the step current being turned on.

Problem 8.4
Modify the computer program used in Exercise 8.4 to demonstrate cavity formation, as illustrated in Fig. 8.18. Apply delayed rate equations using the same device parameters

as in Exercise 8.4, but with a 1 ns round-trip photon delay from a 100% reflecting external cavity mirror as indicated in the following figure.



Problem 8.5

The probability of s discrete photons and n electrons in a laser diode is $P_{n,s}$.

(a) Write down an equation for the time evolution of $P_{n,s}$.

(b) Write a computer program to find $P_{n,s}$ as a function of time and sketch how you expect $P_{n,s}$ to evolve as the injection current in a laser diode is increased.

Problem 8.6

In a bulk semiconductor optical gain medium electron scattering times are short compared to the spontaneous emission time and carriers relax to a quasi-equilibrium described by the Fermi-Dirac distribution, f_{k}. When carrier density, n, is small (and/or temperature, T, is high) such that the chemical potential $\mu \ll E_{\mathrm{F}}$, where E_{F} is the Fermi energy, then f_{k} may be approximated by a Maxwell-Boltzmann distribution. Assuming a reduced electron mass m_{r} such that $\frac{1}{m_{\mathrm{r}}} = \frac{1}{m_{\mathrm{e}}^*} + \frac{1}{m_{\mathrm{hh}}^*}$ where m_{e}^* is the conduction band electron effective mass and m_{hh}^* is the heavy hole effective mass, show that the *total* photon spontaneous emission rate is proportional to n^2 and has temperature dependence $T^{-3/2}$. How is the n^2 dependence of *total* photon spontaneous emission rate modified when n becomes large?

Problem 8.7

Exercise 8.3 illustrates the influence of a Lorentzian broadening function used to model the effect of finite electron-electron scattering time on the optical spontaneous emission and gain spectrum. The spectral broadening $\gamma_{\mathbf{k}}$ is included in the spontaneous emission (Eq. 8.38) and the transformation to optical gain (Eq. 8.33) guarantees optical transparency occurs when photon energy $\hbar\omega$ equals the difference in chemical potential $\Delta\mu$. The parameters are those for GaAs with electron effective mass $m_{\mathrm{e}}^* = 0.07 \times m_0$, heavy hole effective mass $m_{\mathrm{hh}} = 0.5 \times m_0$, carrier density $n = 2 \times 10^{18}\ \mathrm{cm}^{-3}$, absolute temperature $T = 300\ \mathrm{K}$, band-gap energy $E_{\mathrm{g}} = 1.4\ \mathrm{eV}$, $n_{\mathrm{r}} = 3.3$, $\gamma_{\mathbf{k}} = 15\ \mathrm{meV}$, and $g_0 = 2.64 \times 10^4\ \mathrm{cm}^{-1}\ \mathrm{eV}^{-1/2}$.

(a) Reproduce the solution to Exercise 8.3 and on the same scale add a plot of optical gain using Eq. (8.39) with Lorentzian spectral broadening and $\gamma_{\mathbf{k}} = 15\ \mathrm{meV}$.

Comment on the value of optical gain for photon energies $\hbar\omega < E_g$ and when $\hbar\omega = \Delta\mu$.

(b) Add to (a) a plot of optical gain using Eq. (8.33) with a broadening function $\frac{1}{\pi\gamma_{\mathbf{k}}}\operatorname{sech}\left(\frac{E-\hbar\omega}{\gamma_{\mathbf{k}}}\right)$. Comment on the value of optical gain for photon energies $\hbar\omega < E_g$ and when $\hbar\omega = \Delta\mu$.

Problem 8.8

The fourth-order Runge-Kutta method with uniform time step t_{step} can be used to integrate the Langevin single-mode laser diode rate-equations

$$\frac{dS}{dt} = (G-\kappa)S + \beta r_{spon} + F_s(t)$$

$$\frac{dn}{dt} = \frac{I_{inj}}{eV_{vol}} - \frac{n}{\tau_n} - GS + F_e(t)$$

where a normalized Gaussian random number generator (randn in MATLAB) gives

$$F_s(t) = \text{randn} \times \sqrt{\frac{|(G+\kappa)S + \beta r_{spon}|}{t_{step}}}$$

$$F_e(t) = \text{randn} \times \sqrt{\frac{\left|\frac{I_{inj}}{eV_{vol}} + GS + \frac{n}{\tau_n}\right|}{t_{step}}}$$

(a) For a laser diode with the parameters given in Exercise 8.4 and injection current $I_{inj} = 20$ mA, plot relative intensity noise

$$\text{RIN}(f) = \frac{\Delta P(f)}{P_0}$$

as a function of frequency in the range $0.1\text{ GHz} < f < 10\text{ GHz}$. In the expression for RIN, P_0 is the time-average optical power and $\Delta P(f)$ is the average noise power spectral density per unit frequency in the photodetector electrical current.

(b) What is the shot-noise limit of RIN for a laser diode with emission wavelength $\lambda = 1310$ nm and $P_0 = 1$ mW?

Problem 8.9

Continuum mean-field rate equations are used to create Figure 8.17(b) which plots inverse time delay, $1/t_d$, of light emission from a laser diode in response to a step change in current, I_{inj}. The minimum value of $1/t_d$ at laser threshold, I_{th}, is proportional to an energy gap that separates the sustained lasing and non-lasing states of the system. Find experimentally accessible parameters that maximize t_d and hence minimize $1/t_d$. Explain your results and its physical limitations.