Chapter 11 problems

Problem 11.1

Adopting the material parameters and constraints used to generate Fig. 11.2 find a heterostructure diode design that minimizes current density for $0 \leq V_{\text{bias}} \leq 0.2\ \text{V}$ and has current density $J(0.2\ \text{V} < V_{\text{bias}} \leq 0.4\ \text{V}) = (V_{\text{bias}} - 0.2) \times 5 \times 10^{-3}\ \text{A}\ \mu\text{m}^{-2}$. Comment on the feasibility of the design objective and, in particular, what physical phenomena limits device performance.

Problem 11.2

Given integer n_1 indistinguishable photons at port 1 and integer n_2 indistinguishable photons at port 2 of a perfect, lossless, 50:50 dielectric beam-splitter, derive the probability of finding n_3 photons at output port 3 and n_4 photons at output port 4.

Problem 11.3

The quantum field amplitude at input port 1 and input port 2 of a perfect, lossless, 50:50 dielectric beam-splitter is a_1 and a_2 respectively. The quantum field amplitudes at output port 3 and 4 are a_3 and a_4 respectively. The single-photon input amplitudes are related to the output amplitudes via

$$\begin{bmatrix} a_3 \\ a_4 \end{bmatrix}_{\text{out}} = \frac{1}{\sqrt{2}}\begin{bmatrix} \text{i} & -1 \\ -1 & \text{i} \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}_{\text{in}} = \hat{U}_{\text{B}}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}_{\text{in}}$$

where $\hat{U}_{\text{B}}$ is a 2×2 unitary matrix describing the beam splitter.

Find the probability of detecting the photon at output port 3 and the probability of detecting the photon at output port 4 if the single-photon input state is the linear superposition

(a) $|a_1 = \sqrt{1/2}, a_2 = \sqrt{1/2}\rangle$,

(b) $|a_1 = \text{i}\sqrt{1/2}, a_2 = \sqrt{1/2}\rangle$,

(c) $|a_1 = \sqrt{1/4}, a_2 = \sqrt{3/4}\rangle$,

(d) $|a_1 = \text{i}\sqrt{1/4}, a_2 = \sqrt{3/4}\rangle$.

Problem 11.4

Show that Eq. (11.85) may be written as Eq. (11.86).

Problem 11.5

Modify the MATLAB code that created Fig. 11.2, setting maximum voltage $V_{\max} = 0.4\ \text{V}$, maximum current density $J_{\max} = 10^5\ \text{A cm}^{-2}$, and number of barriers $n_{\text{b}} = 16$ to find the following:

(a) A sigmoid current-voltage function centered at $V_{\max}/2$ with zero current at $V_{\text{bias}} = 0$ and current greater than $0.996 \times J_{\max}$ at voltage bias $V_{\text{bias}} = V_{\max}$.

(b) A quadratic current-voltage function with current $J = \frac{J_{\max}}{V_{\max}^2}V_{\text{bias}}^2$.

(c) A polynomial current-voltage function $J = \sum_{n=0}^{4} a_n V_{\text{bias}}^n$ with coefficients $\mathbf{a}^{\text{T}} = [0, 1.8 \times 10^6, -1.09 \times 10^7, 1.85 \times 10^7, -2.63 \times 10^6]$.

(d) A sinusoidal current-voltage function $J = J_{\max}\left(\sin\left(\frac{2\pi V_{\text{bias}}}{V_{\max}}\right)\right)^2$.

(e) How is the quality of the results you obtain for (a)-(d) improved if you use (i) $n_{\text{b}} = 8$ and (ii) $n_{\text{b}} = 32$?

Problem 11.6

(a) Run MATLAB script Chapt11Fig7.m for total input photon number $n_{\text{tot}} = 8$ and $n_{\text{tot}} = 110$. Compare and explain the results.

(b) Calculate average deviation from *zero* for probabilities of observing *odd* numbers of photons exiting output port 3 or 4 when $n_1 = n_{\text{tot}}/2$. Consider $12 \le n_{\text{tot}} \le 128$, with n_{tot} even, and plot average error (deviation from zero) on a log scale as a function of n_{tot} on a linear scale. Comment on how this error varies as a function of n_{tot}.

(c) Repeat part (b), but now use *single* precision to compute the probability amplitude. How does this result differ from double precision?

(d) Using Mathematica and Table 11.3 as a guide, derive simplified analytic expressions of Eq. (11.31) which can be used to accurately calculate the photon detection probabilities in output port 3 for the two extremal cases where (i) $n_1 = n_{\text{tot}}$ and (ii) $n_1 = n_{\text{tot}}/2$. Use these simplified expressions to plot the probability distributions for (i) and (ii) when $n_{\text{tot}} = 1000$ and comment on the results.

Problem 11.7

At time $t = 0$ a coherent photon pulse of unit amplitude, width t_{w}, and spectral peak at wavelength λ_0, is incident normal to the planar dielectric mirrors of a Fabry-Perot resonator with cavity round-trip time $\tau_{\text{RT}} > 2t_{\text{w}}$ and resonant wavelength λ_0. Transmission amplitude is t_{ph} and reflection amplitude is r_{ph} at each ideal, lossless, mirror.

(a) Specify the coherent pulse control sequence (provide values for timing, width, amplitude, phase, and spectrum of each coherent pulse) that will result in *constant* photon energy in the cavity for all times after t_{w}. What is the minimum total energy in the control pulses to achieve this objective?

(b) Specify the coherent pulse control sequence that will result in *constant* photon energy in the cavity whose energy value is half that in (a) for all times after $t_{\text{w}} + \tau_{\text{RT}}/2$.

(c) What limits controllability in (a) and (b)? What limits controllability if the total energy in the system is $\hbar\omega_0 = 2\pi\hbar c/\lambda_0$?

Problem 11.8

Consider a one-dimensional finite chain of *N* atoms with nearest-neighbor tight-binding interaction energy $t_{\text{hop},1}$ and $t_{\text{hop},2}$ that alternates between sites. The *N* sites on the chain are indexed with an integer number *i* starting from site 1 on the left-hand-side. The nearest neighbor separation between sites on the chain is *L* and all on-site potential values V_i are zero. The interaction energy (bond strength) between site 1 and 2 is $t_{\text{hop},1}$. The total number of sites can be even or odd and $t_{\text{hop},1}$ can be greater, equal, or smaller than $t_{\text{hop},2}$.

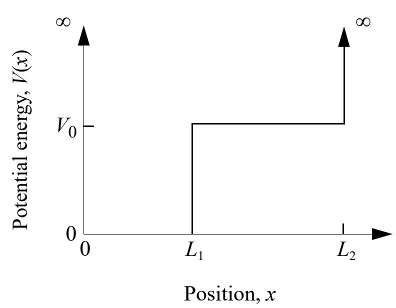
(a) Plot the density of electron states as a function of energy using a Lorentzian eigenenergy broadening of $\gamma = 0.1$ when (i) $N = 8$, $t_{\text{hop},1} = 2$, and $t_{\text{hop},2} = 1$, (ii) $N = 8$, $t_{\text{hop},1} = 1$, and $t_{\text{hop},2} = 2$, (iii) $N = 9$, $t_{\text{hop},1} = 2$, and $t_{\text{hop},2} = 1$, and (iv) $N = 9$, $t_{\text{hop},1} = 1$, and $t_{\text{hop},2} = 2$. Explain the results you obtain. Plot the density of electron states, plot the electron dispersion, and plot the magnitude of the two eigenstates with zero eigenenergy as a function of position on the chain for the case when $N = 100$, $t_{\text{hop},1} = 1$, and $t_{\text{hop},2} = 2$. Explain the results you obtain.

(b) Plot the energy eigenvalues and density of electron states for the bipartite (even value of *N*) case when $N = 100$, $t_{\text{hop},1} = 1$, and $0 \le t_{\text{hop},2} \le 2$. Explain the results you obtain and identify the value of $t_{\text{hop},2}$ at which a phase transition occurs in the thermodynamics limit ($N \to \infty$).

(c) Repeat (b) after introduction of on-site potential $V_1 = -V_N = 0.5$ and after introduction of on-site potential $V_1 = V_N = 0.5$. Explain the results you obtain.

Problem 11.9

An electron is in a one-dimensional piecewise constant potential of thickness L_1 in region 1 and thickness L_2 in region 2. The potential energy in region 1 is $V(0 < x < L_1) = 0$, in region 2 is $V(L_1 \le x < L_2) = V_0$ where V_0 is constant, and the potential is infinite elsewhere.



(a) Write down the form of the wave function in each region if the lowest value of electron eigenenergy is $E = V_0$ and sketch the magnitude of the wave function squared, $|\psi(0 \le x \le L_2)|^2$.

(b) Find the normalization constant for this wave function and determine how it scales as $L_2 \to \infty$.

Problem 11.10

An electron is described by wave function $\psi(x \ge 0) = A e^{-(\gamma/2)x} \sin(k_0 x)$ and is zero elsewhere. For k_0 real valued and γ real and positive, find the expression for the normalization constant $A(k_0, \gamma)$ and determine its value in the limit $\gamma \to 0$. Find the expression for the spectrum $|\psi(k)|^2$ and show that when $\gamma \ll k_0$ this reduces to a Lorentzian function centered at k_0 with full width half maximum γ. Plot the wave function $\psi(x)$, the real and imaginary parts of $\psi(k)$, and the spectrum $|\psi(k)|^2$ for the case when $k_0 = 3.1 \times 10^8\ \text{m}^{-1}$ and the values $\gamma / k_0 = 1$, $\gamma / k_0 = 0.1$, and $\gamma / k_0 = 0.01$. Explain the results you obtain.