

ON THE IRREGULAR FLIGHT OF A TENNIS-BALL.

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It is well known to tennis players that a rapidly rotating ball in moving through the air will often deviate considerably from the vertical plane. There is no difficulty in so projecting a ball against a vertical wall that after rebounding obliquely it shall come back in the air and strike the same wall again. It is sometimes supposed that this phenomena is to be explained as a sort of frictional rolling of the rotating ball on the air condensed in front of it, but the actual deviation is in the opposite direction to that which this explanation supposes. A ball projected horizontally and rotating about a vertical axis, deviates from the vertical plane, as if it were rolling on the air *behind* it. The true explanation was given in general terms many years ago by Prof. Magnus, in a paper "On the Deviation of Projectiles," published in the *Memoirs of the Berlin Academy*, 1852, and translated in Taylor's *Scientific Memoirs*, 1853, p. 210. Instead of supposing the ball to move through air which at a sufficient distance remains undisturbed, it is rather more convenient to transfer the motion to the air, so that a uniform stream impinges on a ball whose centre maintains its position in space—a change not affecting the relative motion on which alone the mutual forces can depend. Under these circumstances, if there be no rotation, the action of the stream, whether there be friction or not, can only give rise to a force in the direction of the stream, having no lateral component. But if the ball rotate, the friction between the solid surface and the adjacent air will generate a sort of whirlpool of rotating air, whose effect may be to modify the force due to the stream. If the rotation take place about an axis perpendicular to the stream, the superposition of the two motions gives rise on the one side to an augmented, and on the other to a diminished velocity, and consequently to a lateral force urging the ball towards that side on which the motions conspire.

The only weak place in this argument is in the last step, in which it is assumed that the pressure is greatest on the side where the velocity is least. The law that a diminished pressure accompanies an increased velocity is only generally true on the assumption that the fluid is frictionless and unacted on by external forces; whereas, in the present case, friction is the immediate cause of the whirlpool motion. The actual mode of generation of the lateral force will be perhaps better understood, if we suppose small vertical blades to project from the surface of the ball. On that side of the ball where the motion of the blades is up stream, their anterior faces are in part exposed to the pressure due to the augmented relative velocity, which pressure necessarily operates also on the contiguous spherical surface of the ball. On the other side the relative motion, and therefore also the lateral pressure, is less; and thus an uncompensated lateral force remains over.

The principal object of the present note is to propose and solve a problem which has sufficient relation to practice to be of interest, while its mathematical conditions are simple enough to allow of an exact solution being obtained. For this purpose I take the case of a cylinder round which a perfect fluid circulates without molecular rotation. At a great distance from the cylinder the fluid is supposed to move with uniform velocity, and the whole motion is in two dimensions. On these suppositions the stream function, on which the whole motion depends, is of the form

$$\psi = \alpha \left(1 - \frac{a^2}{r^2}\right) r \sin \theta + \beta \log r,$$

where r, θ are the polar coordinates of any point of the fluid, measured from the centre of the cylinder and the direction of the stream, as pole and initial line respectively, a is the radius of the cylinder, and α, β are constant coefficients proportional respectively to the velocity of the general current and the velocity of circulation round the cylinder. When $r = a$, ψ is constant, showing that the surface of the cylinder is a stream-line. The radial velocity at any point is given by

$$\frac{d\psi}{r d\theta} = \alpha \left(1 - \frac{a^2}{r^2}\right) \cos \theta;$$

so that, when $r = \infty$ and $\theta = 0$, the radial velocity is α , which is therefore the general velocity of the stream.

At the surface of the cylinder there is no radial velocity, and the magnitude of the tangential velocity is given by

$$d\psi/dr = 2\alpha \sin \theta + \beta/a.$$

Hence, if p_0 be the pressure at a distance, and p the pressure at any point on the surface,

$$2(p - p_0) = \alpha^2 - (2\alpha \sin \theta + \beta/a)^2,$$