A Loop Material Flow System Design for Automated Guided Vehicles

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January 29, 1999

Abstract

We develop an exact integer programming formulation to design a loop material flow system for unit load automated guided vehicles. The model simultaneously determines both the design of the unidirectional loop flow pattern and the location of the pick-up and delivery stations. The objective is to minimize the total loaded vehicle trip distances. To solve the problem, we concentrate on developing a better formulation for the LP sub-problem, pre-processing the problem, identifying the appropriate set of LP/IP routines, analyzing the mathematical properties of the problem, and developing an intelligent branch and bound solution procedure.

Keywords and phrases: Facilities Planning, AGVS, Integer Programming, Network Flows.

1 Introduction

Facilities layout is among the oldest activities of industrial engineers. A good layout always incorporates the design of the material handling system. Estimates of up to a half of the total manufacturing costs are attributed to material handling (Tompkins et al., 1996). Automated guided vehicles (AGVs) are among the modern material handling equipment in manufacturing plants. They are preferred to conveyors due to their flexibility, and to robots due to their mobility.

The design of the material flow system and its simplicity is one of the primary issues in implementing automated guided vehicle systems (AGVS). Blocking is the most undesirable consequence of complicated networks. It results not only in a larger fleet size, but also a throughput below the designed capacity. Furthermore, the software required for dispatching, vehicle routing, and traffic management of complicated networks are quite expensive. Therefore, simple flow patterns have received the most attention in the recent years.

Maxwell and Muckstadt (1982) first introduced the problem of AGV flow system design. While their main concern is vehicle routing, they also addressed material flow path and station location design issues. The flow network they used is known as conventional configuration which is composed of unidirectional arcs. Gaskin and Tanchoco (1987) developed the first integer programming model for material flow path design. Given a fixed network of aisles and fixed pick-up (P) and delivery (D) stations, the model assigns direction to arcs in order to minimize the total trip distances of loaded vehicles. Goets and Egbelu (1990) develop an alternative model where the station locations are no longer fixed but are restricted to the nodes on the boundary of the cells. Sun and Tchernov (1996) provide a comprehensive review on the models developed for conventional configuration.

Afentakis (1989) states the advantages of the loop layout as simplicity and efficiency, low initial and expansion costs, and product and processing flexibility. Loop layout has been studied by many researchers including Bartholdi and Platzman (1989), Sharp and Liu (1990), Kouvelis and Kim (1992), Egbelu (1993), Banerjee and Zhou (1995), and Chang and Egbelu (1996). Bozer and Srinivasan (1989, 1991, and 1994) initiate the concept of tandem configuration as a set of non-overlapping bi-directional loops each with a single vehicle.

The problem discussed in our paper was first conceptualized and modeled by Tanchoco and Sinriech (1992), and Sinriech and Tanchoco (1992 and 1993). The problem is to design a unidirectional loop covering at least one edge of every cell, and to identify the location of the P and D stations on the nodes on each cell. The material handling equipment is a unit load AGV and the objective is to minimize the total loaded vehicle trip distances. Sinriech and Tanchoco (1993) propose a 5-phase serially approach to solve the model. Our goal in this paper is to develop a new formulation and faster solution procedure for the same problem. Our approach differs from their procedure in the following aspects.

1. The number of binary variables required to formulate the degree 2 configuration is reduced by half. Furthermore, the sub-tour elimination approach does not cut any portion of the feasible region of the LP-relaxation containing a feasible solution to the IP problem.

2. We develop a global formulation approach for simultaneous design of both the unidirectional loop flow pattern and the location of the pick-up and delivery stations.

3. To design a solution procedure for the global formulation, we concentrate on developing a better formulation for the LP sub-problem, pre-processing, identifying the appropriate set of LP/IP routines, analyzing the mathematical properties of the problem, and developing an intelligent branch and bound.

We report our results for the prototype example of Sinriech and Tanchoco (1993) which is shown in figure 1, as well as a set of test problems proposed by Nugent et al.(1968). The results show that our formulation is computationally more efficient than the earlier formulations. After formulating the problem in section 2, the theoretical foundations of the LP-relaxation sub-problem are discussed in section 3. Computational considerations are discussed in section 4, and the conclusion follows in section 5.

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Figure 1: The layout and from-to chart of the 11-cell prototype example from Sinriech and Tanchoco (1993).

2 Model Formulation

2.1 Problem Statement

The flow system design problem is defined using the planar graph $G(N, E)$ associated with the block layout. $N = \{n_1, \ldots, n_{|N|}\}$ is the set of intersections on the boundaries of cells. For each pair of adjacent nodes $m, n \in N$ where $n > m$, there is a non-directed edge $mn \in E$. A feasible loop is a circuit containing at least one edge of each cell (Tanchoco and Sinriech, 1992). Not all instances of a block layout contain a feasible loop. Each non-directed edge mn is associated with two directed arcs of $mn, nm \in A$. A symmetric distance function $l : A \rightarrow \Re^+$ assigns l_{mn} as the length of arc mn to be the rectilinear distance between the corresponding nodes on the block layout. Nodes on the boundaries of each face are candidates for station locations. There is one P and one D station per cell. Stations are not necessarily combined.

There is a directed graph $G'(C, F)$ representing the material flow relationships. For each pair of cells $c, k \in C$ with a strictly positive flow from c to k, there is a directed arc $ck \in F$. A function $f : F \rightarrow \Re^+$, assigns f_{ck} as the intensity of material flow to the arc $ck \in F$. The problem is to find a uni-directional loop and its station locations such that the total flow multiplied by the trip distances of the loaded vehicles is minimized. The objective function is stated as,

$$\text{Min } Z = \sum_{c \in C} \sum_{mn \in A} l_{mn} t_{cmn} \tag{1}$$

where t_{cmn} is the decision variable showing the intensity of the total outflow of cell c on arc mn.

2.2 Degree 2 Configuration

Since the distance matrix is symmetric, a degree 2 configuration is formulated by defining a binary variable for each non-directed edge.

$$Y_{mn} \in \{0,1\} \quad \forall mn \in E \tag{2}$$

Y_{mn} is equal to 1, if the non-directed edge mn is on the loop, and 0 otherwise. Given $\mathcal{Y}_c$ as the set of edges on cell c, each cell has at least one edge on the loop.

$$\sum_{mn \in \mathcal{Y}_c} Y_{mn} \geq 1 \qquad \forall c \in C \tag{3}$$

Given any node n, at most 2 of its edges are on the loop.

$$\sum_{m<n} Y_{mn} + \sum_{n<k} Y_{nk} \leq 2 \quad \forall n \in N \tag{4}$$

No node has only one edge on the loop. In other words, at each degree k node, the sum of the decision variables corresponding to each sub-set of $k-1$ edges is greater than or equal to that of the remaining edge.

$$\begin{aligned} \sum_{i<m} Y_{im} + \sum_{\substack{i \neq n \\ m<i}} Y_{mi} \geq Y_{mn} \qquad \forall mn \in E \\ \sum_{\substack{i \neq m \\ i<n}} Y_{in} + \sum_{n<i} Y_{ni} \geq Y_{mn} \qquad \forall mn \in E \end{aligned} \tag{5}$$

2.3 Loop Pattern

The above formulation implies a degree 2 configuration but does not necessarily imply a single loop. Miller et al. (1960) derived the following sub-tour elimination constraint for the Traveling Salesman Problem (TSP).

$$u_m - u_n + |N| X_{mn} \leq |N| - 1 \qquad \forall mn \in A \tag{6}$$

X_{mn} is an integer variable which is 1 if there is a travel from node m to node n and 0 otherwise. u_m is the rank of node n in the sequence of the travel. The advantage of this constraint is its small number. However, it is a weak constraint in general, and in particular it cuts a portion of the integer feasible region of our problem. The constraint becomes stronger if it is modified to

$$u_m - u_n + |N|(X_{mn} + X_{nm}) - 2X_{nm} \leq |N| - 1 \qquad \forall mn \in A \tag{7}$$

But it is still unable to find the loop in a single run. The reason is in the difference between the TSP and the Generalized Traveling Salesman Problem (GTSP). Laporte *et al.* (1996) showed that the problem of finding a loop covering at least one edge of all cells in the block layout is an instance of the GTSP. The above constraint assumes a node to be the first and the last node of the travel. Therefore it is enforced to be on the loop. The nodes covered by the optimal solution of the GTSP are not known in advance. Fixing any node in the solution cuts off a portion of the search space, a portion which may contain the optimal solution. More than one run is required to find the optimal solution for the length of the loop.

Dantzig et al. (1954) derived a sub-tour elimination constraint for the TSP. Their constraint with a slight modification is adopted to our problem as follows.

$$\sum_{m \in R_s} \sum_{n \notin R_s} Y_{mn} + \sum_{n \in R_s} \sum_{m \notin R_s} Y_{nm} \geq 2 \qquad \forall s \in S \tag{8}$$

S is a sub-set of potential adjacent cells. A sub-set of adjacent cells s belongs to S only if formation of the sub-tour on the boundary of s does not ensure formation of sub-tour on the boundary $s' : |s'| < |s|$. R_s is the set of nodes on the cells forming a sub-tour s.

While the constraint directly finds the optimal solution for both TSP and GTSP, its number grows exponentially in the number of cities. Fortunately, the number of these constraints in the block layout is substantially less than that of the general version of GTSP. Furthermore, it requires less integer variables and also creates a tighter LP-relaxation. By pre-processing, Asef-Vaziri *et al* (1998) showed that the average number of required sub-tour elimination constraints for a set of random block layouts of size 30 is less than 800.

2.4 Loop Direction

The direction of the loop is also to be determined by the optimal solution for the total trip distance objective function. A real variable, $0 \leq X_{mn} \leq 1$, is defined for the directed arc mn. The variable is equal to 1 if the directed arc is on the loop, and 0 otherwise. The following constraints state that only one direction is assigned to each edge, and the number of incoming and outgoing arcs at each node are equal.

$$X_{mn} + X_{nm} = Y_{mn} \qquad \forall mn \in E \tag{9}$$

$$\sum_{mn \in A} X_{mn} = \sum_{nm \in A} X_{nm} \quad \forall n \in N \tag{10}$$

Both X_{mn} and X_{nm} are real variables while their corresponding Y_{mn} is integer. However, for a cell with the smallest number of edges, its Y_{mn} variables are left real, while the corresponding X_{mn} variables become integer. Directed arcs of this cell coupled with the arc balance constraint play the interface role to direct the loop. In other words, after branching on edge and arc integer variables, the LP-relaxation of the problem coincides with the IP solutions with respect to the remaining arc variables.

2.5 Stations

Given $\mathcal{N}_c$ as the set of nodes on cell c, a pair of binary variables are defined for each node. The binary decision variable P_{cn} is equal to 1 if node n is selected as the pick-up station of cell c and 0 otherwise. Similarly, D_{cn} is equal to 1 if node n is selected as the delivery station of cell c, and 0 otherwise.

$$P_{cn}, D_{cn} \in \{0, 1\} \quad \forall n \in \mathcal{N}_c \quad \forall c \in C \tag{11}$$

Each cell has one pick-up and one delivery station [1].

$$\sum_{n \in \mathcal{N}_c} P_{cn} = 1 \qquad \forall c \in C \tag{12}$$

[1] Our model can be easily extended to multiple stations per cell. In earlier formulations increasing the number of stations adds a set of new binary variables to the problem. In our formulation, multiple station per cell results in the relaxation of a set of binary variables. Therefore, the solution time is reduced substantially.

$$\sum_{n \in \mathcal{N}_c} D_{cn} = 1 \qquad \forall c \in C \tag{13}$$

2.6 Material Flow

A multi-commodity flow is transferred through the arcs on the loop. We first assume the total outflow of each cell, $f_c = \sum_{ck \in F} f_{ck}$, as a commodity.

$$\sum_{c \in C} t_{cmn} \leq MX_{mn} \qquad \forall mn \in A \tag{14}$$

Flow balance constraints state that the total inflow to a node from all its adjacent nodes and cells is equal to the total outflow from the node to the adjacent nodes and cells[2].

$$P_{cn} \sum_{ck \in F} f_{ck} + \sum_{mn \in A} t_{cmn} = \sum_{ck \in F} f_{ck} D_{kn} + \sum_{nm \in A} t_{cnm} \qquad \forall n \in N \ \forall c \in C \tag{15}$$

Constraints (14) and (15) also play the role of sub-tour elimination. To clarify, if there is a flow between two non-adjacent cells, these constraints ensure that none of the corresponding cells can form a sub-tour. In general, if there is a flow between two sub-tours $s, s' \in S$, they do not need an elimination constraint. By pre-processing it was realized that the total number of required sub-tour elimination constraints for the 11-cell prototype example is 21. This number reduced to 12 when material flow was added to the model. For example, in the prototype example, the sub-tour elimination constraint of a sub-tour corresponding to cell F is required, but the sub-tour elimination constraint for the composite shape formed by cells A and D is not required. Formation of this second sub-tour results in the formation of sub-tour F. Therefore, if sub-tour elimination constraint of F is included in the model, that of combination of A and D is not required. Furthermore, if there exists material flow from cell F to cell A or D, the requirements of material flow prevents formation of both sub-tours F, and combination of A and D. Hence, sub-tour elimination constraints are not needed for cell F.

[2] A node can be a supply point, a demand point, both a supply and demand point, or a transshipment point.

3 The LP-Relaxation

The tightness of the feasible region of the LP-relaxation to the IP solutions plays an important role in the solution time of an integer programming model. In this section the theoretical foundations of the LP-Relaxation of our model are discussed. We prove that many of the integer variables will also be integer in any optimal solution of the LP-relaxation.

3.1 Pick-up Stations

Regarding the properties of pick-up station location variables, we first show that a sub-set of these variables will be integer in the optimal solution to the LP sub-problem.

Theorem 1 *Given a cell c with continuous boundary on the loop, for all nodes n on c but not on k where $f_{ck} > 0$, the value of P_{cn} in the LP-relaxation is either 0 or 1.*

Suppose two nodes i, j with the above properties both have been selected as P stations of cell c. That is, $0 < P_{ci} < 1$ and $0 < P_{cj} < 1$ in the LP-relaxation and $P_{ci} + P_{cj} = 1$. Let $O_c^{(i)}$ be the contribution of cell c in the total outflow $\times$ distance when node i is selected as its P station. The total contribution of cell c in the total (outflow $\times$ distance) in the system is

$$O_c = P_{ci}O_c^{(i)} + P_{cj}O_c^{(j)} \tag{16}$$

Without loss of generality, suppose node j is located after node i on the uni-directional loop on the boundary of cell c, and L_{ij} is the length of the segment from i to j.

$$O_c^{(i)} = O_c^{(j)} + f_c L_{ij}. \tag{17}$$

Therefore,

$$O_c = O_c^{(j)} + P_{ci} f_c L_{ij}. \tag{18}$$

The above expression is minimized, when $P_{ci} = 0$. Therefore, assigning any non-zero value to any node except to node i^* which is the last node of cell c on the directed loop will increase the value of the linear objective function.

$$P_{ci^*} = 1 \qquad P_{ci} = 0 \qquad i \neq i^*. \tag{19}$$

Corollary 1 *When there is a flow from cell c to the adjacent cell(s), the P_{cn} variables corresponding to the common nodes are fractional in the LP-relaxation.*

Suppose there is a flow from cell c to its adjacent cell k, and node i^* is the last node of cell c on the directed loop. By allocating each unit of the outflow of cell c to node i which is in common between c and k, the linear objective function reduces by $f_{ck}L_{i^*i}$, and $P_{ci} = f_{ck}/f_c$ results in $f_{ck}L_{i^*i}$ reduction. The variable P_{ci} does not pass this value unless node i is on the boundary of more than one destination. Otherwise, any additional increase in P_{ci} will increase the objective function by $f_cL_{ii^*}(P_{ci} - f_{ck}/f_c)$.

Corollary 2 *If cell c has a split boundary on the loop, then two of its P_{cn} variables may be fractional.*

Suppose cell c has split boundaries on the unidirectional loop with their last nodes as i and j with respect to the flow direction and the cells boundaries as part of the loop. When node i is selected as a P station of cell c, the total contribution of cell c in the outflow $\times$ distance in the system is

$$O_c = \sum_{k \in KL_{cij}} \sum_{mn \in A_{ij}} t_{ckmn} l_{mn} + L_{ij} \sum_{k \in KL_{cji}} f_{ck} + \sum_{k \in KL_{cji}} \sum_{mn \in A_{ji}} t_{ckmn} l_{mn} \tag{20}$$

NL_{ij} is the set of nodes on the portion of the directed loop from node i to node j. A_{ij} is the set of arcs on the directed loop from node i to j. KL_{cij} is the set of destinations of cell c with their D station on the directed loop from node i to j. Now suppose P_{ci} portion of the outflow of cell c is transferred via node i, and the remaining, P_{cj}, via node j. The total contribution of cell c in the outflow $\times$ distance is

$$O_c = \sum_{k \in KL_{cij}} \sum_{mn \in A_{ij}} t_{ckmn} l_{mn} + L_{ij}(f_c P_{ci} - \sum_{k \in KL_{cij}} f_{ck}) + \sum_{k \in KL_{cji}} \sum_{mn \in A_{ji}} t_{ckmn} l_{mn} + L_{ji}(f_c P_{cj} - \sum_{k \in KL_{cji}} f_{ck}) \tag{21}$$

The above expression is minimized when

$$f_c P_{ci} = \sum_{k \in KL_{cij}} f_{ck} \Rightarrow P_{ci} = \sum_{k \in KL_{cij}} f_{ck}/f_c \tag{22}$$

$$f_c P_{cj} = \sum_{k \in KL_{cji}} f_{ck} \Rightarrow P_{cj} = \sum_{k \in KL_{cji}} f_{ck}/f_c \tag{23}$$

and the total contribution of the outflow of cell c is

$$O_c = \sum_{k \in KL_{cij}} \sum_{mn \in A_{ij}} t_{ckmn} l_{mn} + \sum_{k \in KL_{cji}} \sum_{mn \in A_{ji}} t_{ckmn} l_{mn}. \tag{24}$$

3.2 Delivery Stations

We next show that under certain conditions all the decision variables corresponding to the delivery station locations will be binary in the optimal solution to the LP-relaxation.

Theorem 2 *The LP-relaxation of the problem coincides with the IP solutions with respect to all the D_{kn} variables when there is an unique node in cell k with minimum contribution to the objective function.*

Suppose node i is selected as a D station of cell k. The total contribution of the inflow of cell k in the objective function is

$$I_k^i = \sum_{mn \in A} l_{mn} \sum_{ck \in F} t_{ckmn} \tag{25}$$

If cell k splits its inflow among its candidate D stations, its contribution in the objective function is equal to

$$I_k = \sum_{i \in \mathcal{N}_k} D_{ki} I_k^i. \tag{26}$$

Let,

$$I_k^* = Min \ \{I_k^i\}. \tag{27}$$

Then

$$I_k = \sum_{i \in \mathcal{N}_k} D_{ki}(I_k^i - I_k^* + I_k^*), \tag{28}$$

which equals,

$$I_k = I_k^* + \sum_{\substack{i \in \mathcal{N}_k \\ i \neq i^*}} D_{ki}(I_k^i - I_k^*). \tag{29}$$

The above expression is minimum when

$$D_{ki} = 0 \qquad \forall i \neq i^* \tag{30}$$

Therefore, each destination cell in order to minimize its contribution in the objective function will have only one D station. The only exception is when there is a node $i \neq i^*$ where $I_k^i = I_k^*$. As corollary 3 shows, this situation is extremely rare.

Corollary 3 *If cell k has two nodes i, j on its boundary such that*

$$\sum_{c \in CL_{kij}} f_{ck}/L_{ij} = \sum_{ck \in F} f_{ck}/(L_{ij} + L_{ji}), \tag{31}$$

then there is more than one node satisfying $I_k^i = I_k^*$. That is, the LP optimal values for D_{ki} and D_{kj} variables are not necessarily integer.

CL_{kij} is the set of cells having a non-zero flow with cell k, and their pick-up station located on the path ij on the directed loop.

If node i is assigned as the delivery station of cell k, the inflow contribution of this cell in the objective function is

$$I_k^i = \sum_{c \in CL_{kij}} \sum_{mn \in A_{ij}} t_{cmn} l_{mn} + \sum_{c \in CL_{kji}} \sum_{mn \in A_{ji}} t_{cmn} l_{mn} + \Big(\sum_{c \in CL_{kij}} f_{ck}\Big)\Big(\sum_{mn \in A_{ji}} l_{mn}\Big) \tag{32}$$

Similarly, when node j is selected as the D station of cell k, its contribution in the total flow is

$$I_k^j = \sum_{c \in CL_{kji}} \sum_{mn \in A_{ji}} t_{cmn} l_{mn} + \sum_{c \in CL_{kij}} \sum_{mn \in A_{ij}} t_{cmn} l_{mn} + \Big(\sum_{c \in CL_{kji}} f_{ck}\Big)\Big(\sum_{mn \in A_{ij}} l_{mn}\Big) \tag{33}$$

The model is indifferent between the nodes i, j if $I_k^i = I_k^j = I_k^*$. Since the sum of the two first terms in I_k^i and I_k^j are equal, their third terms have to be equal

$$\Big(\sum_{c \in CL_{kij}} f_{ck}\Big)\Big(\sum_{mn \in A_{ji}} l_{mn}\Big) = \Big(\sum_{c \in CL_{kji}} f_{ck}\Big)\Big(\sum_{mn \in A_{ij}} l_{mn}\Big) \tag{34}$$

and in a straight forward manipulation

$$\sum_{c \in CL_{kij}} f_{kc}/L_{ij} = \sum_{ck \in F} f_{ck}/(L_{ij} + L_{ji}) \tag{35}$$

The above equation states that the total inflow of cell k divided by the length of the loop is equal to its inflow from the cells with their P stations between i and j divided by the length of the directed loop in this interval.

Corollary 4 *Given cell c with only one outflow which is to its adjacent cell k having more than one inflow, then the P_{cn} variables will be integer in the optimal solution of the LP-relaxation.*

As stated in theorem 1, cell c may split its outflow among its common nodes with cell k. However, based on theorem 2 the destination cell will have only one D station. Therefore, the optimal values for the P variables of the origin cell will be integer.

We remark that using the above theorems and corollaries reduces the number of P and D station location variables that require branching in the 11-cell prototype example from 86 to 15.

3.3 Edges

The appropriate value of M in constraint (14) plays an important role in both the validity of the integer solution and tightness of the LP-relaxation of edges. If M is very large, even when X_{mn} is very small, their multiplication could be greater than $\sum t_{cmn}$. In a LP/IP solver, depending on the coefficients of the model and the precision factor of the software, small values are assumed zero by the software. Therefore, the optimal integer solution may contain some flows passing both directions of an edge which is not even on the loop.

The second issue is that the feasible region of the LP-relaxation becomes closer to integer solutions, and also the branches have a higher chance to go on the right direction as M gets smaller. Note that the value of Y_{mn} in the LP-relaxation is

$$Y_{mn} = \sum_{c \in C} (t_{cmn} + t_{cnm})/M$$

If M is unnecessary large, then most probably Y_{mn} is branched on 0 while a strictly positive flow is passing it. An upper bound for M is the total flow in the system, $\sum_{ck \in F} f_{ck}$. Furthermore, increasing the number of constraints and replacing them by the following constraints results in a better LP relaxation for edge variables.

$$t_{cmn} \leq (\sum_{ck \in F} f_{ck}) X_{mn} \qquad \forall mn \in A \quad \forall c \in C \tag{36}$$

In the present form of constraints (15) and (36) , the total outflow of each row of the FT-chart is assumed as a commodity. Each flow has one origin and one or more destinations. There are $|C| \times |N|$ sets of multi-commodity flow balance constraints. As stated earlier, except for a sub-set of the P stations, all station variables are integer in the optimal solution of the LP-relaxation.

In an alternative formulation, each element of the FT-chart is defined as a commodity. Therefore, the number of multi-commodity flow balance constraints is increased to $|F| \times |N|$. The flow balance constraint (15) is replaced by the following

$$f_{ck} P_{cn} + \sum_{mn \in A} t_{ckmn} = f_{ck} D_{kn} + \sum_{nm \in A} t_{cknm} \qquad \forall n \in N \ \ \forall ck \in F \tag{37}$$

where t_{ckmn} is the decision variable showing the intensity of flow ck on arc mn. In this new formulation, all station variables will be integer in the optimal solution of the LP-relaxation. Again, in the expense of having more constraints, constraint (36) is replaced by

$$t_{ckmn} \leq f_{ck} X_{mn} \quad \forall mn \in A \quad \forall ck \in F \tag{38}$$

which pushes the edge variables closer to 0 or 1 in the LP-relaxation.

3.4 Variable and Value Ordering

The fundamental insight gained in the previous sections is implemented to develop an intelligent branch and bound solution procedure. The variable ordering for branching has a substantial impact on the efficiency of the solution procedure. In the taxonomy of variables, the highest branching priority is assigned to edges, second to arcs, third to P stations, and last to D stations. At each node of the branch and bound search, as long as the value of an edge variable is 0, its corresponding arc variable values are both set to 0. If the values of all edges incident to a node are 0, then all station variables corresponding to that node are set to 0. After branching on each layer of the variables, the LP-relaxation for the next layer either coincides or is very close to the binary values. When the branch and bound search approaches the next layer of variables, a sub-set of them are already binary. They do not require branching. The remaining variables are close to either 0 or 1. The first branch is usually on the right direction. Such a variable ordering is almost the same as removing some variables from the set of integers. The perfect instantiation of this situation in our problem are the station location variables. After branching on edges and a few arcs, a majority of the P station variables become 0 or 1. After branching on the remaining P stations, almost all D station variables are already 0 or 1.

Within the layer of edges, they are further classified based on their potential to be on the loop. Regarding every flow $f_{ck} > 0$, if c and k are adjacent, f_{ck} is added to the weight of the common nodes, otherwise it is added to the weight of all their nodes. The node priority vector is defined as a permutation of integers $1, \cdots, |N|$ denoted by $P = P_1, \cdots, P_{|N|}$; where P_i is the node with the ith highest weight. The priority vector of nodes in the prototype example is 3,5,15,11,6,12,16,7,8, 2,18,9,4, 14,13,10,17,1. Out of the first 10 nodes, 8 of them are on the optimal loop.

The priority of nodes is translated into the priority of edges. To explain the procedure, nodes 3 and 5 have the first and second priorities. Edge 3-5 does not exist. The third node is 15. However, 3-15 and 5-15 do not also exist. The fourth node is 11, and edge 3-11 gets the first priority for branching. The edges are grouped into 5 classes of priority. Edges with higher priorities are not only branched on first, but are first branched on the value of 1.

4 Computational Considerations

In this section we first compare our global approach with the serial approach by Sinriech and Tanchoco (1993), and then report additional computational results. The serial approach is composed of 5 phases. Phase-1 employs an integer program or alternatively a heuristic to find a loop covering at least one edge of each cell. Phase-2 starts from the obtained loop and enumerates all feasible loops. Phase-3 applies three rules to drop inferior loops and sub-tours. Phase-4 applies a mixed integer program on each of the remaining loops to find the optimal location of stations on that loop. The loop with the minimum loaded vehicle trip distance is identified in phase-5. Although the authors report no computation time, we attempt to find a basis for comparison.

Sinriech and Tanchoco (1992) report the computation times to find the location of the P and D stations on a fixed loop. The model accounts for both inter-cell and intra-cell material handling. However, it becomes identical to phase-4 if inter-cell material handling cost is set to 0. The model is solved for 4 different loops, each fixed on the layout of the 11-cell prototype example and using the same FT chart given in figure 1 (b). It takes 10 seconds to 20 minutes on a GOULD NP 1 using a modified version of CPLEX to find the optimal location of the stations on each fixed loop. As stated in Sinriech and Tanchoco (1993), in the 11-cell example, out of the 444 loops enumerated in phase-2, there are 17 loop left after phase-3. The optimal solution for the 11-cell example using our global approach is shown in figure 2 (a)[3]. All computations are using CPLEX 4 on a Sun Enterprise 4000/5000. The solution time for the case of assuming the total outflow of each cell as a commodity was 7 seconds, plus .5 second for preprocessing for enumeration and screening of sub-tours. The value of the objective function at the root node was 43 percent of its optimal integer solution. By assuming each element of the FT chart as a commodity, the LP objective function was lifted to 80 percent of the IP value. As a result, the solution

[3] Thickness of edges represent the intensity of trips on the edge.

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Figure 2: The optimal solutions for the prototype example.

figure=LP.eps

Figure 3: Efficiency of LP/IP routines applied on the prototype layout example coupled with 25 and 50 percent dense randomly generated FT charts.

time reduced to 1.6 seconds.

The intensity of flow in the prototype example by Tanchoco and Sinriech (1992) was 15 percent. To further examine the model, flows were generated from the uniform distribution of 0 to 100. Two sets of FT charts, one set with density of 25 percent and the other set with 50 percent density were filled with these flows. Figure 3 shows CPU time comparison for different LP/IP routines implemented to solve the problems. Since the matrix of coefficient is tall and the primal problem is highly degenerate, the dual simplex outperforms the primal. Indeed, the dual performs better than both the primal and network simplex in the root node as well as in the sub-sequent nodes. Therefore, the dual simplex is implemented in the remaining experiments. The steepest edge pricing performed better than all other dual gradient pricing rules. Although depth first search immediately identifies a feasible integer solution and continuously improves it, the quality of the subsequent solutions are not as well as that of the other backtracking strategies. Indeed, it is dominated by both best linear lower bound and best estimated integer objective function when all integer infeasibilities are removed. Sometimes, it may be expected to stop the program before it finds the optimal solution or prove its optimality. In this case, the node selection strategy of best linear lower bound and variable selection strategy of pseudo reduced costs is recommended. There will be a higher chance to have a better feasible solution after a specific amount of time, or a solution of specific quality in a shorter amount of time. Finally the variable ordering as well as value ordering figure=cpu.eps

Figure 4: Statistics for the problems of coupling the prototype layout example with 25 and 50 percent dense randomly generated FT charts.

realized the last significant reduction in CPU times. Figures 3 and 4 show the average of the statistics for the corresponding sets of problems.

To examine the model on larger problems, we expanded the prototype FT chart of Figure 1 into a 20-cell FT chart. This expanded problem can be found in Nugent et al. (1968). The expansion rule was $f_{(10+c)(10+k)} = f_{ck} \ \forall ck \in F$, and any index of 21 is replaced by 1. The new 20-cell FT chart which contains 38 flows was applied on the six 20-cell test layouts of Nugent et al. (1968). The average number of rows, columns, and non-zero coefficients were 5130, 3831, and 17000 respectively. The average number of iterations and nodes required to find the optimal solution were 178608 and 1561, respectively. The CPU time was 851 seconds. The average quality of the LP objective function at the root node and that of the first integer solution were around 65 and 150 percent of the optimal solution, respectively. Given a 50, 20, and 10 percent difference between the integer solution and the linear lower bound, an integer solution of a quality of 25, 10, and 5 percent deviation from optimal solution were obtained.

5 Conclusions

We formulated the simultaneous loop flow pattern design and pickup and station location problems as an integer programming model. One particular advantage of our formulation is that many of the integer variables are also binary in the optimal solution of the LP-relaxation, hence greatly improving the solution time of the integer program. We have also presented search strategies that improve the computational efficiency of the branch and bound procedure. In an experimental section, we showed the computational efficiency of our global approach on well-known problem sets in the literature. The direction of our future research is to develop a loop based partition for a tandem AGV system.

From a practitioner point of view, the user simply needs to have access to a mixed-integer program solver such as CPLEX to implement our solution procedure. The inputs to our model are a block layout in which each cell is defined by a set of connected horizontal and vertical lines, and the from-to material flow chart. The model is given by Equations 1-5, 8-13, and 37-38. Finally, the settings for the CPLEX solver should be as follows: (1) dual simplex for LP-relaxtion problem, (2) steepest edge for dual gradient pricing, (3) best integer lower bound for the backtracking strategy, and (4) implement variable and value ordering as given Section 3.4.

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Appendix

D_{cn}	:	The binary variable corresponding to node n as the delivery station of cell c
f_{ck}	:	Intensity of the flow from cell c to cell k
F	:	Set of strictly positive flows
l_{mn}	:	Rectilinear distance from node m to node n
N	:	Set of nodes
P_{cn}	:	The binary variable corresponding to node n as the pick up station of cell c
t_{ckmn}	:	The proportion of the flow from cell c to cell k passing arc mn
t_{cmn}	:	The proportion of the total flow of cell c passing arc mn
X_{mn}	:	The binary variable corresponding to the directed edge mn
Y_{mn}	:	The binary variable corresponding to the non-directed edge mn where $n > m$